Math 60 8.7 Absolute Value Equations and Inequalities
(2 days.)

Equations ↓ $=$ Inequalities ↓ $<, >, \le, \ge$

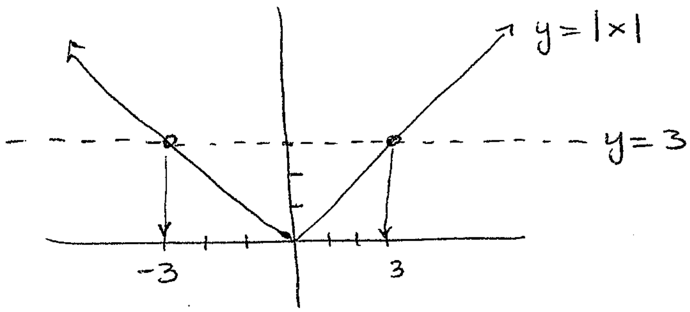
Objectives 1) Solve absolute value equations containing one absolute value.

2) Solve absolute value equations containing two absolute values.

GOAL: Solve $|x| = 3$.

Explore:

① Graph $y = |x|$, use to find solutions of $|x| = 3$.

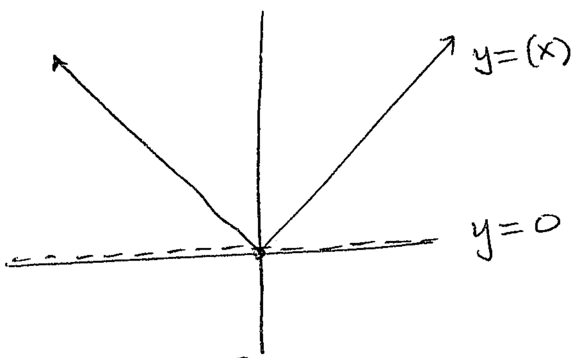


The equation $|x| = 3$ is $3 = |x|$ or when $y=3$ on graph of $y = |x|$.

The line $y=3$ intersects the graph $y=|x|$ twice $\boxed{x=3}$ and $\boxed{x=-3}$.

Absolute value equations often have two solutions.

② Use graph of $y=|x|$ to solve $|x| = 0$



$\boxed{x=0}$

It's possible for absolute value equations to have only one solution.

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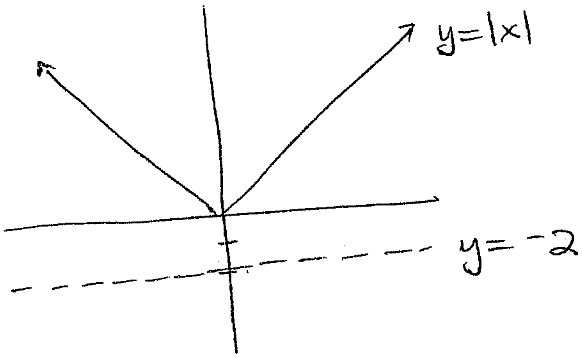
③ Use graph of $y=|x|$ to solve $|x|=-2$



The line $y=-2$ and the graph $y=|x|$ do not intersect

$\boxed{\text{no solution}}$

So it is possible for an absolute value equation to have two, one, or no solutions.

④ Solve $|x|=-6$

$\boxed{\text{no solution}}$

The result from an absolute value cannot be a negative number.

⑤ Solve $|2x-3|=-5$

$\boxed{\text{no solution}}$

Even if it's the absolute value of a more complex expression, the result cannot be negative.

⑥ Solve $|x|=3$

step 1: write two equations by
- removing absolute values
- using + RHS for one equation
- using − RHS for the second equation

$\boxed{x=3}$ $\boxed{x=-3}$

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⑦ Solve $|2x+3| = 6$

Step 1: Write two equations.

$2x+3=6$ or $2x+3=-6$

Step 2: Isolate the variable

$$\frac{2x}{2} = \frac{3}{2} \qquad \frac{2x}{2} = -\frac{9}{2}$$

$$\boxed{x = \frac{3}{2}} \qquad \boxed{x = \frac{-9}{2}}$$

Step 3: MathXL will request a solution set

$$\boxed{\left\{\frac{3}{2}, \frac{-9}{2}\right\}}$$

Either form of the answer is acceptable on a quiz or exam.

⑧ Solve $|2x+3| - 1 = 6$

Step 0: Isolate the absolute value first.

$$|2x+3| = 7$$

Step 1: Write two equations

$$2x+3=7 \quad \text{or} \quad 2x+3=-7$$
$$\underline{-3 \quad -3} \qquad \underline{-3 \quad -3}$$

$$\frac{2x}{2} = \frac{4}{2} \qquad \frac{2x}{2} = \frac{-10}{2}$$

$$\boxed{x=2} \qquad \boxed{x=-5}$$

or $\boxed{\{2, -5\}}$

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(9) Solve $|x| = -2$

$\boxed{\text{no solution}}$

If you were sound asleep:

$x = -2$ or $x = -(-2)$

$x = -2$ $\quad$ $x = 2$

check by plugging in:

$|-2| \stackrel{?}{=} -2$ $\quad$ $|2| \stackrel{?}{=} -2$

$2 \neq -2$ $\quad$ $2 \neq -2$

both answers are extraneous.

$\boxed{\text{no solution}}$

(10) Solve $|3-x| + 2 = 1$

step 0: Isolate the absolute value.

$|3-x| = -1$

$\boxed{\text{no solution}}$

(11) Solve $-2|1-3x| + 1 = -3$

step 0: Isolate the absolute value

$$\frac{-2|1-3x|}{-2} = \frac{-4}{-2}$$

$$|1-3x| = 2$$

step 1: Write two equations (then solve)

$1 - 3x = 2$ $\quad$ $1 - 3x = -2$

$-1 \quad\quad -1$ $\quad$ $-1 \quad\quad -1$

$\frac{-3x}{-3} = \frac{1}{-3}$ $\quad$ $\frac{-3x}{-3} = \frac{-3}{-3}$

$\boxed{x = -\frac{1}{3}}$ $\quad$ $\boxed{x = 1}$

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(12)

$$|4-z| - z = 2$$

$$|4-z| = z+2$$

Step 1: Isolate absolute value (add z both sides)

step 2
write two equations for absolute value

$$4-z = z+2 \qquad\qquad 4-z = -(z+2)$$

caution: Must use ( )
Entire RHS is opposite.

step 3: Solve each equation.

$$4-z = z+2$$
$$\underline{+z} \quad \underline{+z}$$

collect z's together

$$4 = 2z+2$$
$$\underline{-2} \quad \underline{-2}$$

isolate z

$$\frac{2}{2} = \frac{2z}{2}$$

$$\underline{\underline{z=1}}$$

$$4-z = -(z+2)$$

dist neg

$$4-z = -z-2$$

collect z's

$$\underline{+z} \quad \underline{+z}$$

$$4 \neq -2$$

contradiction means no solution from this part

$\boxed{z=1}$ is the only solution

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13 Solve

$$|x+1| = |2x+3|$$

LONG WAY: (Don't do this)

Write two equations for absolute value on right

(A) $|x+1| = 2x+3$ or (B) $-|x+1| = 2x+3$

mult/divide by

$|x+1| = -(2x+3)$

Write two equations for each absolute value

(A) $x+1 = 2x+3$ or $x+1 = -(2x+3)$

(B) $x+1 = -(2x+3)$ or $x+1 = -[-(2x+3)]$

NOTICE: WE GET ONLY TWO DIFFERENT OPTIONS.

$x+1 = 2x+3$ or $x+1 = -(2x+3)$

Better method! continued →

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SHORT WAY: (Do this)

step 1: Remove both absolute values

Write two equations — one RHS unchanged

— one w/ dist -1 to RHS.

$x+1 = 2x+3$ or $x+1 = -(2x+3)$

step 2: Isolate variable in each.

$$\begin{aligned} x+1 &= 2x+3 \\ -x\quad & \quad -x \\ 1 &= x+3 \\ -3\quad & \quad -3 \end{aligned}$$

$\boxed{-2=x}$

$$\begin{aligned} x+1 &= -2x-3 \\ +2x\quad & \quad +2x \\ 3x+1 &= -3 \\ -1\quad & \quad -1 \\ \frac{3x}{3} &= \frac{-4}{3} \end{aligned}$$

$\boxed{x=-4/3}$

step 3: (Optional) Check by substituting back

$x=-2$:

$|-2+1| \stackrel{?}{=} |2(-2)+3|$

$|-1| \stackrel{?}{=} |-4+3|$

$1 = |-1|$

✓

$x = -4/3$

$|-4/3+1| = |2(-4/3)+3|$

$|-1/3| = |-8/3+3|$

$1/3 = |1/3|$

✓

(14) $|2x-3| = |5-2x|$

$$\begin{aligned} 2x-3 &= 5-2x \\ +2x\quad & \quad +2x \\ 4x-3 &= 5 \\ +3\quad & \quad +3 \\ \frac{4x}{4} &= \frac{8}{4} \\ x &= 2 \end{aligned}$$

or

$$\begin{aligned} 2x-3 &= -(5-2x) \\ 2x-3 &= -5+2x \\ -2x\quad & \quad -2x \\ 0-3 &= -5+0 \\ -3 &= -5 \end{aligned}$$

no solution

$\boxed{x=2}$ is the only solution

Practice: Solve.

(15) $|2x+5|+7=3$

Isolate absolute value

$|2x+5|=-4$

wide awake:

$\boxed{\text{no solution}}$

(16) $|x+1|+3=3$

$|x+1|=0$

$x+1=0$ $\qquad$ $x+1=-0$

$\boxed{x=-1}$ $\qquad$ same $\qquad$ → This question has only one solution.

(17) $|-5x+2|-2=5$

$\qquad\qquad +2 \quad +2$

$|-5x+2|=7$

$-5x+2=7$ $\quad$ or $\quad$ $-5x+2=-7$

$\quad -2 \quad -2 \qquad\qquad\qquad -2 \quad -2$

$\dfrac{-5x}{-5}=\dfrac{5}{-5}$ $\qquad$ $\dfrac{-5x}{-5}=\dfrac{-9}{-5}$

$\boxed{x=-1}$ $\qquad$ $\boxed{x=\frac{9}{5}}$

If you get only one of the two answers you have not done the problem correctly.

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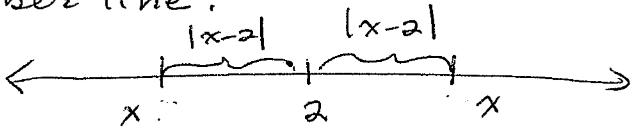
Geometrical explanation #2

Another way to think about the same question:

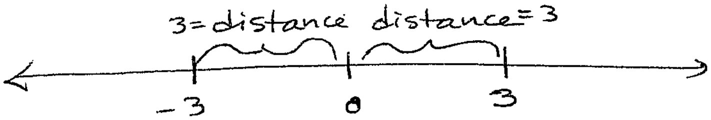
Remember that $|x-2|$ is the distance x is from 2 on the number line.



To solve

$$|x| = 3$$

We realize $|x| = |x-0|$ is distance from 0.



If the distance from x to 0 is 3 units,

x could be -3 (3 units left of 0)

or x could be $+3$ (3 units right of 0).